x^3

Chapter (5) Equations, inequality and graphs

$-x^3$

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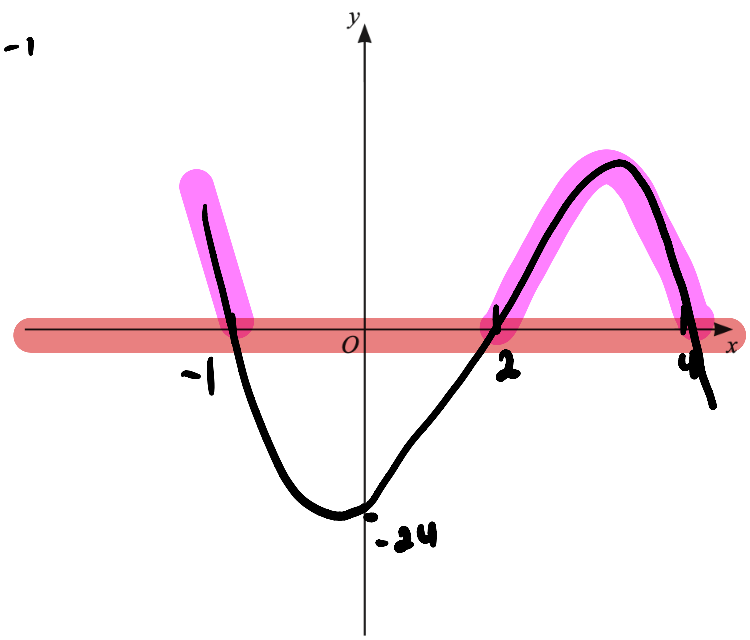
1. (a) On the axes below, sketch the graph of $y = -3(x-2)(x-4)(x+1)$, showing the coordinates of the points where the curve intersects the coordinate axes.

$x = 0$, $y = -3 \times (-2)(-4)(1)$
$= -24$

[3]

$y = 0$, $x = 2, 4, -1$



(b) Hence find the values of *x* for which $-3(x-2)(x-4)(x+1) > 0$.

[2]

$x < -1$, $2 < x < 4$

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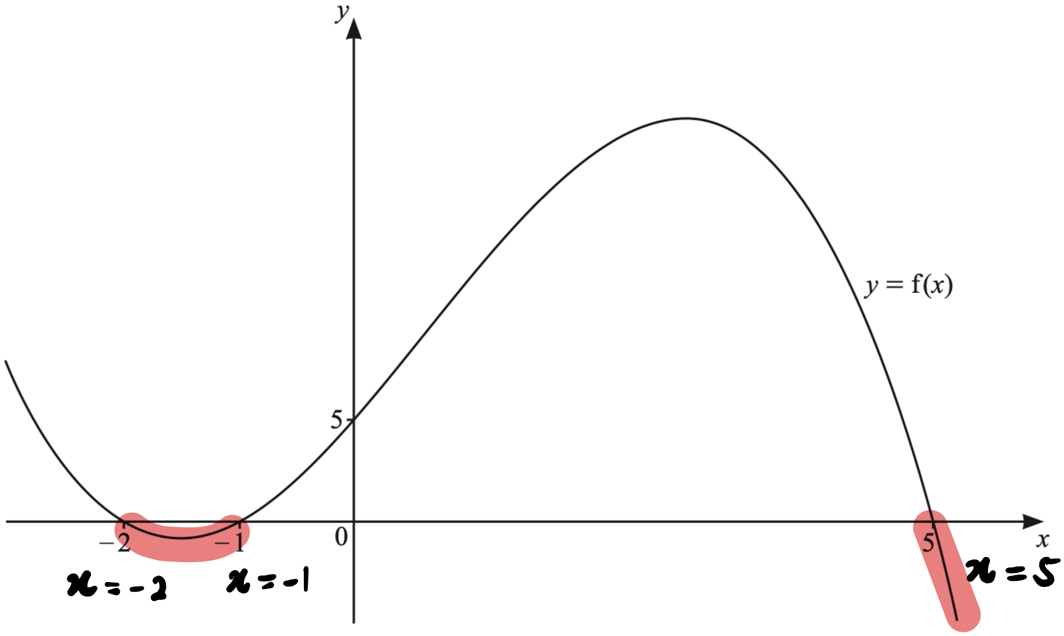
2. The diagram shows the graph of a cubic curve $y = f(x)$.



$2 \times 1 \times -5$

$= -10$

(a) Find an expression for $f(x)$.

$y = -\frac{1}{2}x(x+2)(x+1)(x-5)$

[2]

(b) Solve $f(x) \le 0$.

[2]

$-2 \leq x \leq -1 \quad , x \geq 5$

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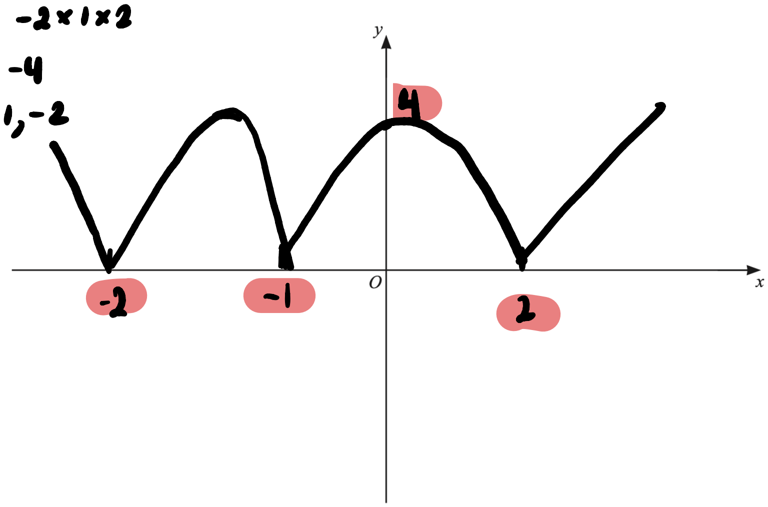
3. On the axes below, sketch the graph of $y = |(x - 2)(x + 1)(x + 2)|$ showing the coordinates of the points where the curve meets the axes.

[3]

$x = 0,\ y = -2 \times 1 \times 2$
$= -4$
$y = 0,\ x = 2, -1, -2$



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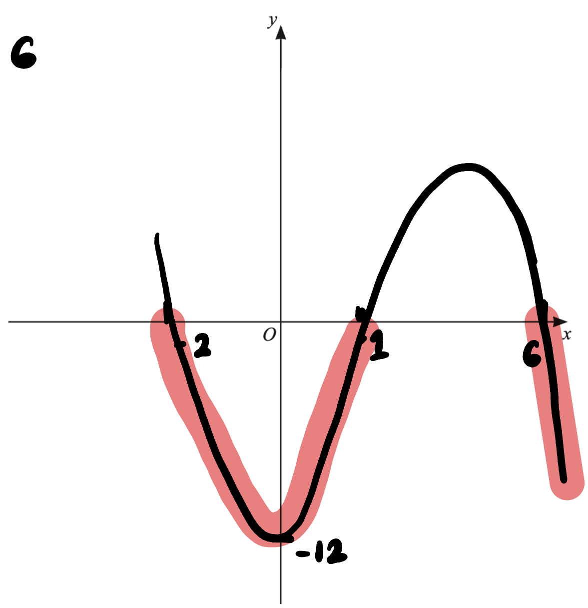
4. (a) On the axes below, sketch the graph of $y =- (x + 2)(x - 1)(x - 6)$, showing the coordinates of the points where the graph meets the coordinate axes.

[2]

$x = 0,\ y = -1 \times 2 \times -1 \times -6$
$= -12$
$y = 0,\ x = -2, 1, 6$



(b) Hence solve $-(x + 2)(x - 1)(x - 6) \leq 0$.

[2]

$-2 \leq x \leq 1,\ x \geq 6$

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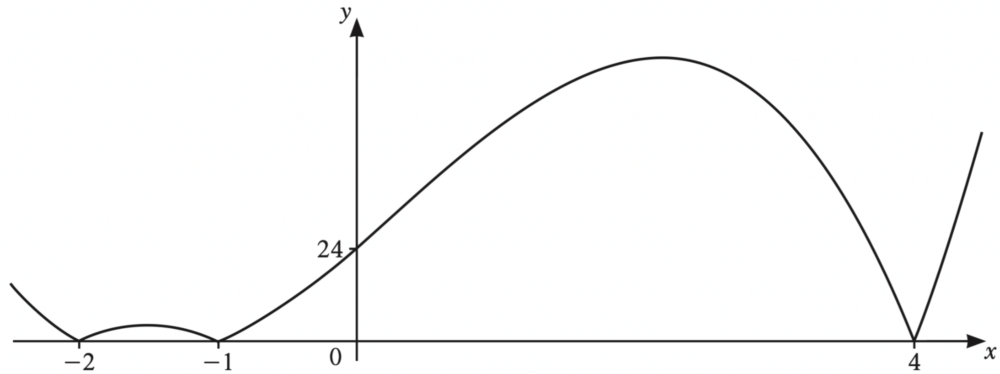
5.



2 × 1 × −4
= −8 × 3
= −24

The diagram shows the graph of $y = |p(x)|$ where $p(x)$ is a cubic function. Find the two possible expressions for $p(x)$.

[3]

$p(x) = -3(x+2)(x+1)(x-4)$

or

$p(x) = 3(x+2)(x+1)(x-4)$

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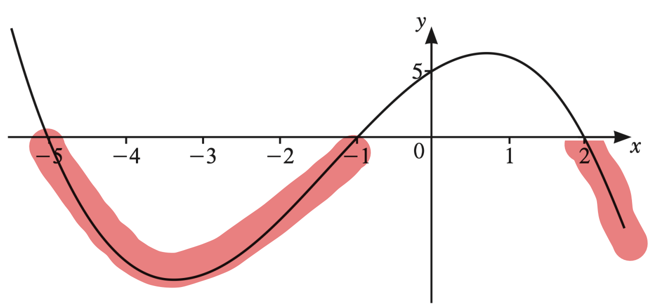
6.



The diagram shows the graph of $y = f(x)$, where $f(x)$ is a cubic polynomial.

(a) Find $f(x)$.

$f(x) = -\frac{1}{2}(x+5)(x+1)(x-2)$

5 × 1 × −2 = −10 ÷ −2
= 5

[3]

(b) Write down the values of x such that $f(x) < 0$.

$-5 < x < -1, \quad x > 2$

[2]

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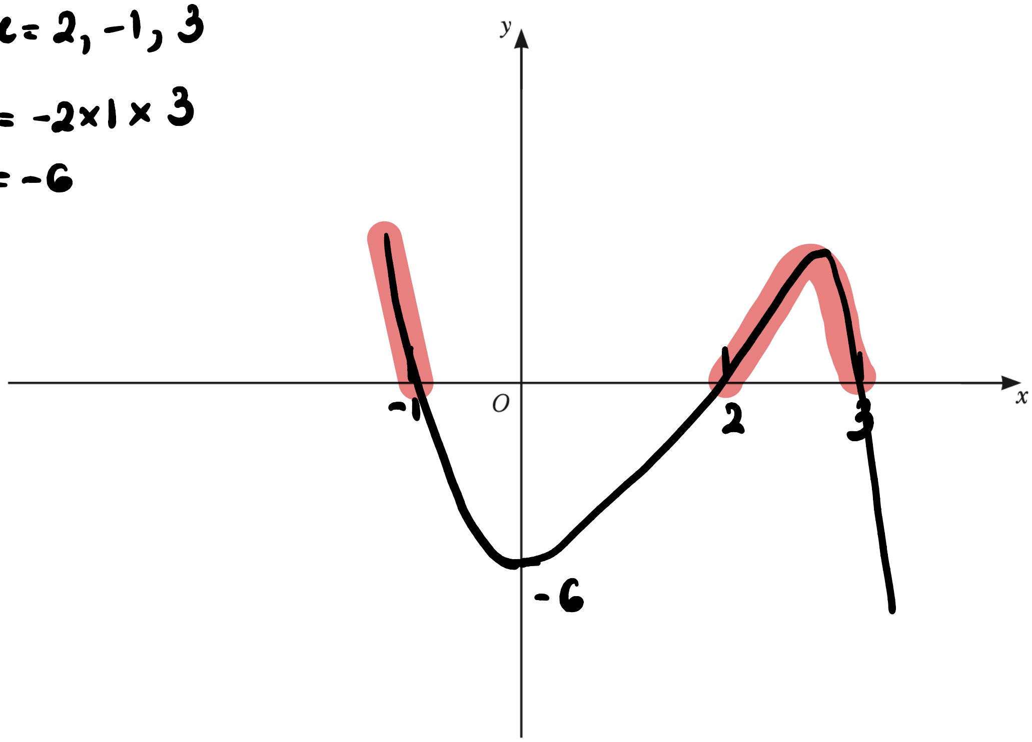
7.(a) On the axes below, sketch the graph of $y = (x - 2)(x + 1)(3 - x)$ stating the intercepts on the coordinate axes.

[3]

$y = 0,\ x = 2, -1, 3$

$x = 0,\ y = -2 \times 1 \times 3$

$= -6$



(b) Hence write down the values of x such that $(x - 2)(x + 1)(3 - x) > 0$.

[2]

$x < -1$ or $2 < x < 3$

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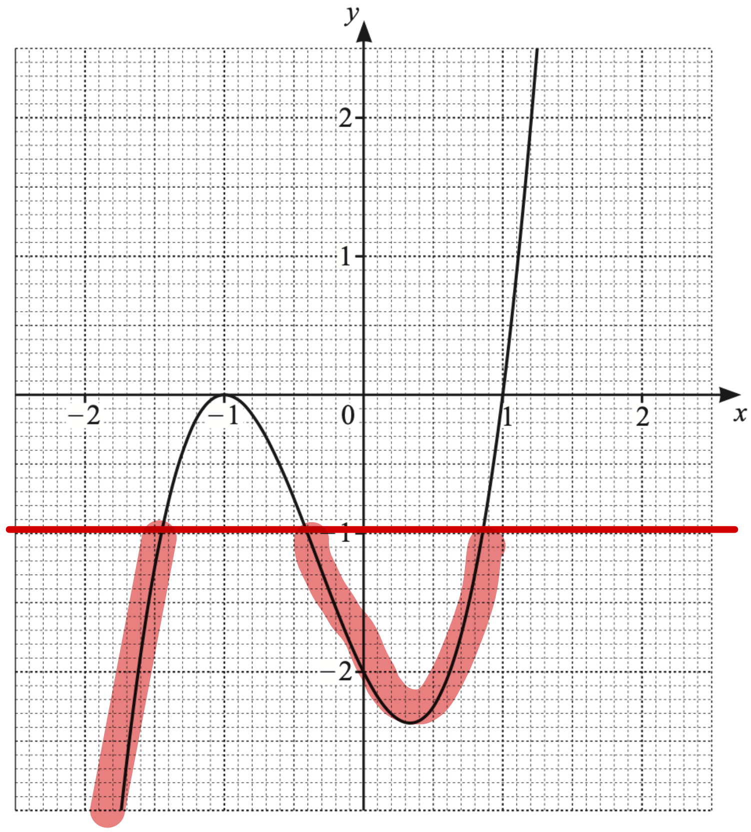
8.



The diagram shows the graph of $y = f(x)$, where $f(x) = a(x + b)^2(x + c)$ and *a*, *b* and *c* are integers.

(a) Find the value of each of *a*, *b* and *c*. [2]

$1 \times 1 \times -1$

$= -1$

$f(x) = 2(x+1)(x+1)(x-1)$

$a = 2, \quad b = 1, \quad c = -1$

(b) Hence solve the inequality $f(x) \leq -1$. [3]

$x \leq -1.4 \qquad -0.4 \leq x \leq 0.9$

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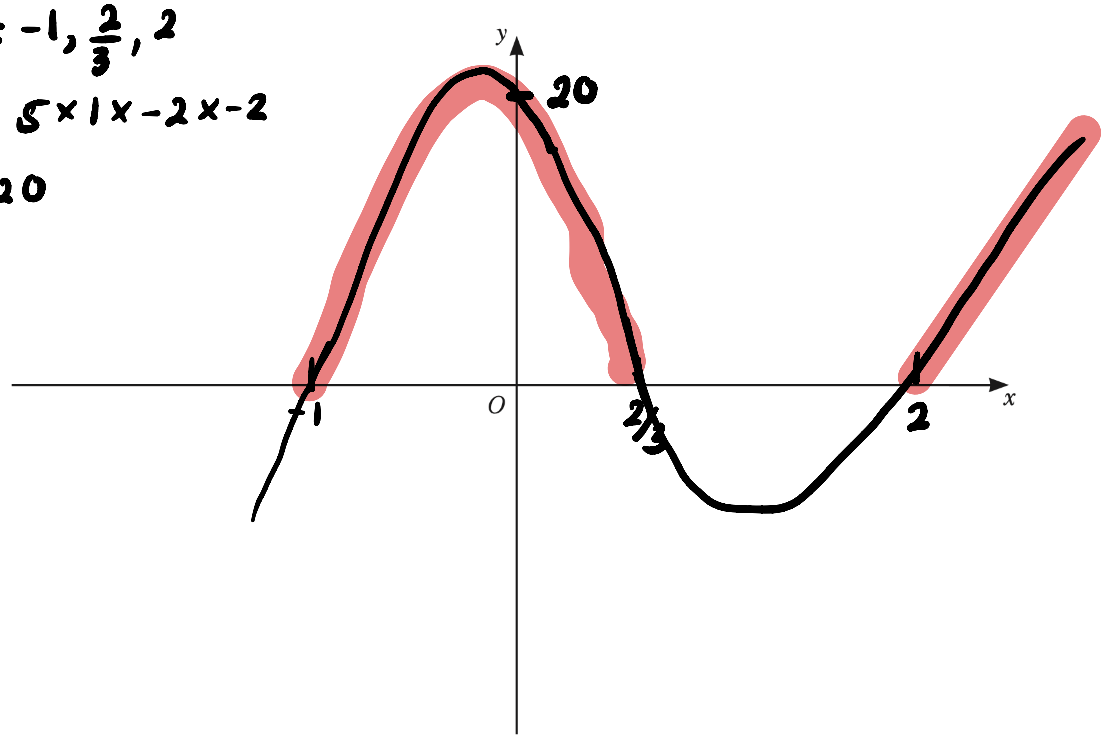
9. (a) On the axes, sketch the graph of $y = 5(x + 1)(3x - 2)(x - 2)$, stating the intercepts with the coordinate axes.

$y=0,\ x = -1, \frac{2}{3}, 2$

$x=0,\ y = 5 \times 1 \times -2 \times -2$

$= 20$

[3]



(b) Hence find the values of x for which $5(x + 1)(3x - 2)(x - 2) > 0$.

$-1 < x < \frac{2}{3}$ $\quad x > 2$

[2]

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$x = 1/5$

10. The three roots of $p(x) = 0$ where $p(x) = 5x^3 + ax^2 + bx - 2$ are ~~$x = 1/5$~~, $x = n$ and $x = n + 1$, where *a* and *b* are positive integers and *n* is a negative integer. Find p(*x*), simplifying your coefficients.

[5]

$(5x - 1)(x - n)(x - n - 1)$

$-n-1$

$= 2-1$

$= 1$

$-1 \times -n \times (-n-1)$

$= +n \times (-n-1)$

$-2 = -n^2 - n$

$n^2 + n - 2 = 0$

$n = 1$ (Reject) or $n = -2$

$(5x-1)(x+2)(x+1)$

$= (5x^2 + 10x - x - 2)(x+1)$

$= (5x^2 + 9x - 2)(x+1)$

$= 5x^3 + 5x^2 + 9x^2 + 9x - 2x - 2$

$= 5x^3 + 14x^2 + 7x - 2$

$a = 14, b = 7$

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